DYNAMIC STRESSES IN TRANSIENTLY LOADED PARABOLIC SHELLS

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INTRODUCTION

A group of problems has been determined [1] on the analysis of the strength properties of the parabolic lens of a focusing device [2] for a neutrino experiment, constituting two paraboloid shells of revolution connected at the vertices by a structural pin. The magnetic pressure on the outer surface of the shells arising as a transient electric current passing through the lens leads to the appearance of dynamic stresses in them. The stress-deformed state of such shells has been obtained [1, 3] in a static approximation. In this work, the dynamic stresses of a parabolic lens are investigated for current pulses constituting a single sinusoid half-period lasting between 70 and 300 μsec at an amplitude of 500 kA. This dependence is a good approximation to the actual shape of current pulses following at intervals between 6 and 8 sec. Since the decay time of free mechanical vibrations of shells is significantly less than the pulse interval, the initial conditions for the arrival of the next pulse are assumed to be zero. A variant of a rigid fastening of the lens (fastening the paraboloid shell with a pin and flange) is considered as the most acceptable in accordance with a previously conducted analysis [1]. The purpose of our study is to determine the maximal dynamic stresses for subsequent estimates of the fatigue strength. Since in this case the duration of the period of the lower tones of the natural modes of the shells is comparable to the duration of a current pulse, a complete solution of the problem, consisting in investigating the stressed-deformed state both during the passage of the current and in terms of the natural modes of the lens following the pulse, is necessary. Transient loading of a shell of variable thickness by a spatially inhomogeneous pressure is a rather complex problem. It is therefore numerically solved using two methods in order to insure completeness and reliability of the results. These methods are the eigenfunction method using the matrix-fitting method for integration of the differential equations, and the method of nets [4]. The results are analyzed. The fundamental notations in the work are analogous to those of [1, 3].

The system of differential equations describing the dynamic stressed-deformed state of a lens treated as a thin-walled shell of revolution loaded with internal pressure is described in the form

$$\frac{\rho h}{\rho_0 h_0}\tilde{E}\frac{\partial^2 \mathbf{X}(\bar{s},\tau)}{\partial \tau^2} = F(\bar{s})\mathbf{X}(\bar{s},\tau) - \frac{\partial \mathbf{X}(\bar{s},\tau)}{\partial \bar{s}} - \mathbf{G}(\bar{s},\tau), \tag{1}$$

where $\mathbf{X}(\bar{s},\tau)=\{\bar{u},\bar{w},\theta_1,\bar{T}_1,\bar{Q}_1,\bar{M}_1\}$ is the vector of the desired functions, $\mathbf{F}(\bar{s})$ is a square (6 × 6) coefficient matrix, $\mathbf{G}(\bar{s},\tau)=[0,0,0,0-\bar{p}_n,0]$ is the load function vector, $\tau=(t/R_0)\sqrt{B_0/\rho_0 h_0}$ is the dimensionless time parameter, ρ is the density of the material at an arbitrary point of the shell, ρ_0 is the density at a reference point, and $\tilde{E}$ is a square matrix in which

$$\tilde{\mathbf{E}}_{21} = \begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix},$$

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the remaining minors ($\tilde{E}_{11}$, $\tilde{E}_{12}$, $\tilde{E}_{22}$) being zero.

In this problem, $G(\bar{s}, \tau)=G_0(\bar{s})q(\tau)=G_0(\bar{s}) \sin^2 \Omega t$, where Ω is the angular frequency of variation of the current passing through the lens. Thus the solution of the transient loading problem for a lens reduces to the solution of a nonhomogeneous system of differential equations (1) under zero initial at two-point boundary conditions.

The solution (1) is usually represented in the eigenfunction method [5-7] in the form

$$\mathbf{X}(\bar{s}, \tau) = \sum_n \mathbf{X}_n(\bar{s})\, \varphi_n(\tau),$$

where $\mathbf{X}_n(\bar{s})$ are the eigenfunctions of the homogeneous system (1) [$G(\bar{s}, \tau)=0$] corresponding to the natural modes $\bar{\omega}_n=\omega_n R_0 \sqrt{\rho_0 h_0/B_0}$. This representation leads in this case to slowly converging series. A significant improvement in the convergence can be obtained by using as a first approximation of the vector, the functions

$$\mathbf{X}_0(\bar{s}, \tau)=\mathbf{X}_0(\bar{s})q(\tau),$$

where $\mathbf{X}_0(\bar{s})$ is the static solution of the system of Eqs. (1) at $G(\bar{s}, \tau)\equiv G_0(\bar{s})$. Only the minor $\mathbf{X}_0(\bar{s}, \tau)$ complementary to the exact solution is represented in the form of a series in eigenfunctions.

To find the natural modes and shapes of the shell vibrations, the system of homogeneous ordinary differential equations obtained from Eq. (1) by replacing $\partial^2/\partial\tau^2$ by $-\bar{\omega}^2$ is solved by the two-sided matrix-fitting method [1, 8], i.e., by finding the rigidity matrix L or yielding matrix $\tilde{L}$, inverse to it for a series of values of $\bar{\omega}$. The natural mode corresponds to the case when the determinant of the yielding matrix vanishes as we pass from one end of the shell to the other. It corresponds to the presence of trivial solution in the case of a rigid fastening. The given mode $\bar{\omega}$ may turn out to be a natural mode for a part of the shell truncated from $\bar{s}_0$ to $\bar{s}$, corresponding to a free edge in the course of integrating at a given point $\bar{s}_1$ and may correspond to a fastening at a second point $\bar{s}_2$. In the first case, this leads to the matrix $\tilde{L}$, becoming infinite, and in the second case, it may reduce to the matrix L. The passage through this point is carried out by inverting the increasing matrix and continuing integration of the equations for the inverse matrix. Thus the method of calculation may be nonuniquely varied in the course of integration, more frequently, the greater is the desired mode. This fact limits the number of natural modes that may be determined, though it may turn out to be completely sufficient for practical purposes (for example, 60 fundamental modes were determined for one lens).

The calculations demonstrated that the series converge at a sufficient rate within a wide range of durations of the current pulse, if we use the above method and isolate the quasistatic component $\mathbf{X}_0(\bar{s}, \tau)$.

The algorithm for solving the problem by the method of nets consists in finding $\mathbf{X}(\bar{s}, \tau_0+\Delta\tau)$ using the system of Eqs. (1) if $\mathbf{X}(\bar{s}, \tau)$ and $\partial \mathbf{X}(\bar{s}, \tau)/\partial\tau$ are known at $\tau=\tau_0$, and $\bar{s}_0<\bar{s}<\bar{s}_k$. The value of $\partial \mathbf{X}(\bar{s}, \tau)/\partial\bar{s}$ is represented in the form of finite differences, boundary conditions being used on the edge of the shell. Integration with respect to time is carried out by the Runge–Kutta method. The integral of integration with respect to time was limited by an abruptly appearing instability of the solution for fixed values of steps Δt with respect to time and coordinate Δs, that satisfied the physical condition $c\cdot\Delta t<\Delta s$, where c is the speed of sound in the shell material. We need only decrease Δs and Δt to expand the integral, the solutions differing only insignificantly in the stability region. The latter, as well as the abrupt nature under which the instability appears, can be reliably referred to the calculation results.

The high ($\sim$2%) qualitative agreement between the maximal stresses and the values of the components $\mathbf{X}(\bar{s}, \tau)$ along the lens meridian obtained by the two methods indicates that our calculations were correct.

Table 1 presents the geometric parameters of the three types of lens of a focusing device for which calculations were carried out; a is the constant of the parabola generating the shell, r_0 is the pin radius, r_k is the radius of the outer surface at the flange, L is the length of the lens, and h_0 is the shell thickness along lines of constant radii. A

TABLE 1

Number of lens	a, cm^{-1}	r_0, cm	r_k, cm	h_0, cm	σ_{st}, kg/mm^2	f_1, kHz	σ_d, kg/mm^2
1	0,842	1,2	6,5	1,5	14,85	7,10	18,0
2	0,404	1,5	9,5	1,0	15,5	6,30	18,5
3	0,1236	3,0	24,0	1,0	8,3	2,82	7,2

TABLE 2

n	1	2	3	4	5	6	7	8
$\bar{\omega}_n$	0,09132	0,13256	0,13747	0,14475	0,14833	0,15510	0,15960	0,1674
f_n, kHz	6,30	9,17	9,49	10,0	10,25	10,7	11,02	11,56
n	9	10	11	12	13	14	15	16
$\bar{\omega}_n$	0,17413	0,18266	0,19066	0,20023	0,20907	0,21854	0,22210	0,23266
f_n, kHz	12,02	12,59	13,15	13,82	14,44	15,11	15,35	16,08

series of fundamental modes and the shapes of the natural modes of shells, the distribution of the dynamic stresses, and displacements along the meridian length s for a pulse duration of 157 μsec ($\Omega = 2\cdot 10^4 \sec^{-1}$), were determined and the dependence of the maximal stresses on pulse duration was investigated.

The fundamental natural modes f_1 are presented in Table 1 for the three lenses, while the natural modes f_n and dimensionless angular frequencies $\bar{\omega}_n$ for lens 2 are given in Table 2, column 16. The fundamental modes f_1 were determined by the longitudinal vibrations and were close to c/L. A high density of the modes beginning with the second mode is a distinctive feature of the spectrum.

The shapes of the natural modes with respect to $\bar{u}$ (curves 1) and $\bar{\omega}$ (curves 2) of lens 2 for the first (a), second (b), third (c), and seventh (d) tones are shown in Fig. 1. Excitation of the fundamental tones leads to the appearance of displacements U sufficiently uniformly along the entire length, whereas the radial vibrations, the chief contribution to which are provided by the displacements $\bar{w}$, are excited only near the flange, i.e., along the greater segment of the shell. The appearance of radial vibrations near the pin, on the other hand, is possible only with generation of sufficiently high tones.

Figure 2 illustrates the evolution of the stress distributions and σ_{M_2} for the lens 2 (curves 1-4, respectively). Distributions are presented for the following moments of time after the start of the pulse: a) 50 μsec, b) 100 μsec; and c) 200 μsec.

The nature of the distributions for the selected currrent pulse parameters in its maximum region (100 μsec) is nearly static [1, 3]. As in the static case, equivalent stresses $\sigma_1 = \sigma_{T1} + \sigma_{M1}$ are greatest on the outer surface of the paraboloid near the pin. Figure 3 depicts their variation in time for lens 2 (curve 1). The shape of the current pulse is depicted here by curve 2. The maximum of the equivalent stresses lies within the limits of the pulse duration, but lag with respect to the current maximum. The amplitude σ_1 for free lens vibrations following the end of the current is roughly one-half.

Table 1 presents the maximal equivalent dynamic σ_d and static σ_{st} stresses for all the lens at $\Omega = 2\cdot 10^4 \sec^{-1}$ and I_0 = 500 kA. We have $2\sigma_d > \sigma_{st}$ for lens 1 and $\sigma_d < \sigma_{st}$ for lens 3. This is because the frequency Ω/π of the induced force is close in the first case to the frequency f_1 of the primary tone of the natural modes and is twice the latter in the second case. The dependence of the maximal equivalent stress on the current pulse duration and σ_d is reached at $\Omega/\pi \simeq f_1$. The σ_d tends to σ_{st} with increasing current pulse duration

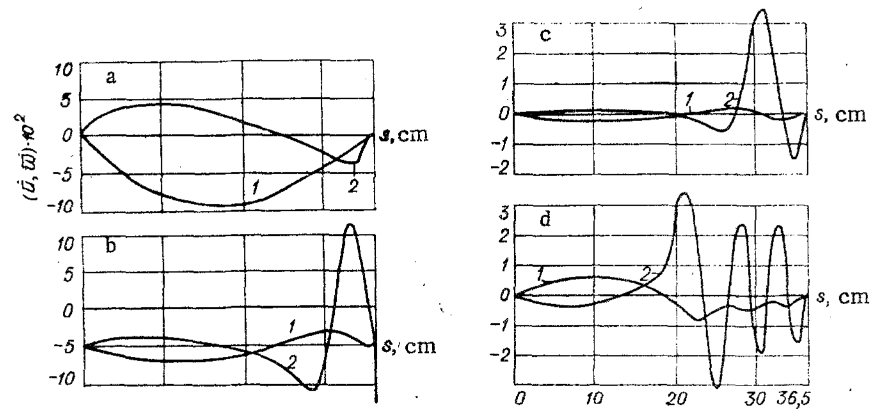

Fig. 1

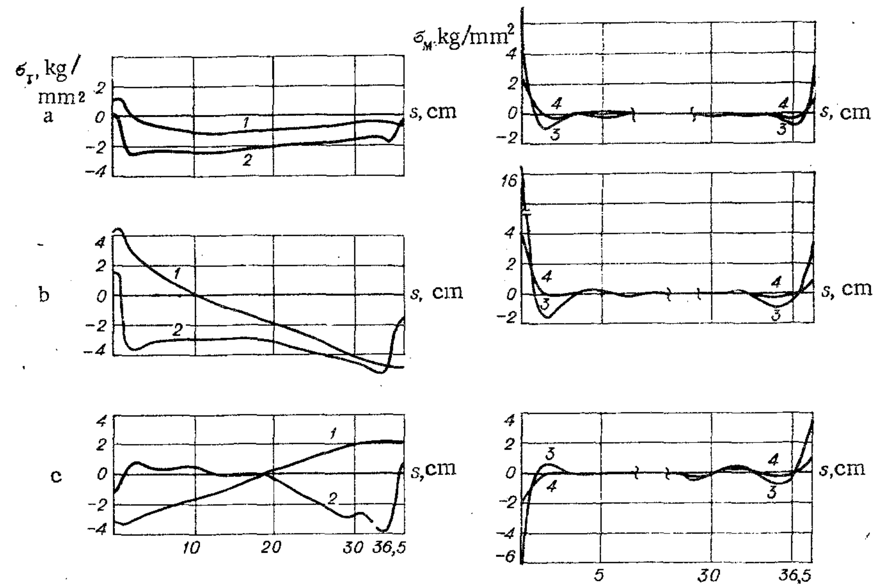

Fig. 2

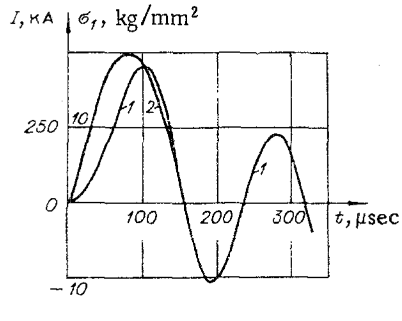

Fig. 3

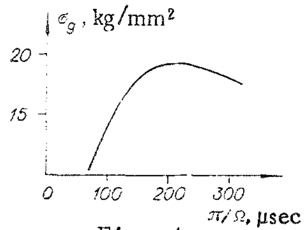

Fig. 4

and σ_d decreases, approaching a dependence proportional to $\pi f_1/\Omega$, with decreasing stress duration.

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